UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO
FACULTAD DE INGENIERÍA
DIVISIÓN DE CIENCIAS BÁSICAS
COORDINACIÓN DE MATEMÁTICAS

CÁLCULO INTEGRAL
TERCER EXAMEN EXTRAORDINARIO

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3 de junio de 2019 — *Semestre 2019-2*

INSTRUCCIONES: Lee cuidadosamente los enunciados de los **6 reactivos** que componen el examen antes de empezar a resolverlos. La duración máxima del examen es de **2 horas**.

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1. Obtén la serie de Taylor de la función

$$f(x) = e^{2x}$$

alrededor de $a = 1$

15 puntos

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2. Determina si la siguiente integral converge o diverge

$$\int_{-1}^{3} \frac{dx}{\sqrt{1 + x}}$$

15 puntos

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3. Efectúa:

$a)\ \int \frac{dx}{x^2(x-1)}$ $\qquad$ $b)\ \int \frac{x}{\sqrt{1+x^4}}\,dx$ $\qquad$ $c)\ \int \operatorname{ang\,tan} x\ dx$

30 puntos

4. Calcula la longitud de arco de la gráfica de la función $f(x) = 2\sqrt{x^3}$ en el intervalo $\left[0,\ \frac{1}{3}\right]$

10 puntos

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5. Sean $f(u, v) = u^2 - v$ y $u = e^{2x-y}$, $v = \ln(xy)$, calcula

$$\left.\frac{\partial f}{\partial y}\right|_{(1,\ 1)}$$

15 puntos

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6. Obtén la ecuación cartesiana del plano tangente a la gráfica de la función $z = -x^2 - y^2 + 4$ en el punto $P(1,\ 1)$

15 puntos

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1221
Solución del Tercer Examen Extraordinario
Semestre 2019 – 2

1. Sea:

$$f(x)=e^{2x} \quad ; \quad a=1$$

$$f(x)=f(a)+f'(a)(x-a)+\frac{1}{2!}f''(a)(x-a)^2+\frac{1}{3!}f'''(a)(x-a)^3+\ldots$$

$$f(x)=e^{2x} \qquad f(1)=e^2$$

$$f'(x)=2e^{2x} \qquad f'(1)=2e^2$$

$$f''(x)=4e^{2x} \qquad f''(1)=4e^2$$

$$f'''(x)=8e^{2x} \qquad f'''(1)=8e^2$$

$$\vdots$$

$$e^{2x}=e^2+2e^2(x-1)+\frac{1}{2!}4e^2(x-1)^2+\frac{1}{3!}8e^2(x-1)^3+\ldots$$

$$\boxed{e^{2x}=e^2+2e^2(x-1)+2e^2(x-1)^2+\frac{4}{3}e^2(x-1)^3+\ldots}$$

15 puntos

2. Es impropia:

$$\int_{-1}^{3}\frac{dx}{\sqrt{1+x}} \qquad 1+x>0\,;\,x>-1 \quad \text{No continua en } x=-1$$

$$\int_{-1}^{3}\frac{dx}{\sqrt{1+x}}=\lim_{\varepsilon\to 0^+}\left(\int_{-1+\varepsilon}^{3}\frac{dx}{\sqrt{1+x}}\right)$$

$$Si \quad I=\int \frac{dx}{\sqrt{1+x}} \qquad \left| \begin{array}{l} u=1+x \\ du=dx \end{array} \right.$$

$$I=\int \frac{du}{u^{1/2}}=\int u^{-1/2}du=\frac{u^{1/2}}{1/2}+C=2u^{1/2}+C$$

$$I=2\sqrt{1+x}+C$$

$$\Rightarrow \int_{-1}^{3} \frac{dx}{\sqrt{1+x}}=\lim_{\varepsilon\to 0^+}\left(\left[2\sqrt{1+x}\right]_{-1+\varepsilon}^{3}\right)$$

$$=\lim_{\varepsilon\to 0^+} 2\left(\sqrt{1+3}-\sqrt{1-1+\varepsilon}\right)$$

$$=\lim_{\varepsilon\to 0^+} 2\left(2-\sqrt{\varepsilon}\right)=4$$

$$\boxed{\int_{-1}^{3} \frac{dx}{\sqrt{1+x}}=4 \quad \therefore \quad \text{Convergente}}$$

15 Puntos

3. Solución

a) Por descomposición en fracciones parciales

Sea $\dfrac{1}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$;

$$1 = Ax(x-1) + B(x-1) + Cx^2$$

$$1 = A(x^2 - x) + B(x-1) + Cx^2$$

$$\begin{aligned} A + \quad C &= 0 \qquad & \boxed{A=-1} \\ -A + B \quad &= 0 & \boxed{B=-1} \\ -B \quad &= 1 & \boxed{C=1} \end{aligned}$$

$$\frac{1}{x^2(x-1)} = -\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1}$$

$$I = \int \left(-\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x-1} \right) dx = -\int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{dx}{x-1}$$

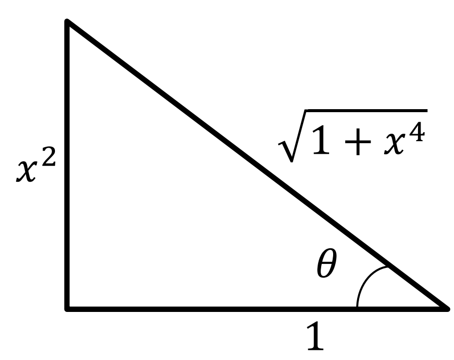
$$I = -ln(x) + \frac{1}{x} + ln(x-1) + C \quad \Rightarrow \boxed{I = ln\left(\frac{x-1}{x}\right) + \frac{1}{x} + C}$$

b) Por sustitución trigonométrica

$$x^2 = tan(\theta)$$

$$2xdx = sec^2(\theta)d\theta; \quad xdx = \frac{1}{2}sec^2(\theta)d\theta$$

$$\sqrt{1+x^4} = sec(\theta)$$



Al sustituir en la integral:

$$I = \frac{1}{2}\int \frac{sec^2(\theta)d\theta}{sec(\theta)} = \frac{1}{2}\int sec(\theta)d\theta$$

$$I = \frac{1}{2}ln\big(tan(\theta) + sec(\theta)\big) + C \Rightarrow \boxed{I = ln\left(\sqrt{1+x^4} + x^2\right)^{1/2} + +C}$$

c) Por partes

$$\left| \begin{array}{l} u = ang\,tan(x) \\ du = \dfrac{dx}{1+x^2} \end{array} \right. \Rightarrow \left| \begin{array}{l} dv = dx \\ v = x \end{array} \right.$$

$$I = x\,ang\,tan(x) - \int \frac{x\,dx}{1+x^2}$$

$$\omega = 1 + x^2$$

$$d\omega = 2xdx$$

$$xdx = \frac{1}{2}d\omega$$

$$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{d\omega}{\omega} = \frac{1}{2}ln(\omega) + C$$

$$\int \frac{x\,dx}{1+x^2} = \frac{1}{2}ln\left(1+x^2\right) + C$$

$$\boxed{I = x\,ang\,tan(x) - \frac{1}{2}ln\left(1+x^2\right) + C}$$

30 Puntos

4.

$$f(x) = 2\sqrt{x^3} = 2x^{3/2} \quad ; \quad \left[0, \frac{1}{3}\right]$$

$$L = \int_a^b \sqrt{1 + \left[f'(x)\right]^2} dx$$

$$f'(x) = 2 \cdot \frac{3}{2} x^{1/2} = 3x^{1/2} \quad \therefore \quad \left[f'(x)\right]^2 = 9x$$

$$L = \int_0^{1/3} \sqrt{1 + 9x} dx$$

$$u = 1 + 9x$$

$$du = 9dx$$

$$dx = \frac{1}{9} du$$

$$\int \sqrt{1 + 9x} dx = \frac{1}{9} \int u^{1/2} du = \frac{1}{9} \left[\frac{u^{3/2}}{3/2} \right] + C$$

$$L = \frac{2}{27} \left[(1 + 9x)^{3/2} \right]_0^{1/3} = \frac{2}{27} \left[\left(1 + 9\left(\frac{1}{3}\right)\right)^{3/2} - 1 \right]$$

$$L = \frac{2}{27} [7]$$

$$\boxed{L = \frac{14}{27} \ u}$$

10 Puntos

5.

$$f = u^2 - v \quad ; \quad u = e^{2x-y} \quad ; \quad v = ln(xy)$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,1)} = ?$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$\frac{\partial f}{\partial u} = 2u; \quad \frac{\partial f}{\partial v} = -1; \quad \frac{\partial u}{\partial y} = -e^{2x-y}; \quad \frac{\partial v}{\partial y} = \frac{x}{xy} = \frac{1}{y}$$

$$\frac{\partial f}{\partial y} = 2u\cdot\left(-e^{2x-y}\right) + (-1)\cdot\frac{1}{y}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{y} - 2u\left(e^{2x-y}\right) = -\frac{1}{y} - 2\left(e^{2x-y}\right)\left(e^{2x-y}\right)$$

$$\frac{\partial f}{\partial y} = -\frac{1}{y} - 2\left(e^{2x-y}\right)^2$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,1)} = -\frac{1}{1} - 2\left(e^{2(1)-1}\right)^2 = \boxed{-1-2e^2}$$

15 Puntos

6.

$$z = -x^2 - y^2 + 4; \quad P(1,1,2)$$

$$-(x - x_0)\frac{\partial z}{\partial x} - (y - y_0)\frac{\partial z}{\partial y} + (z - z_0) = 0$$

$$\frac{\partial z}{\partial x} = -2x; \quad \left.\frac{\partial z}{\partial x}\right|_P = -2$$

$$\frac{\partial z}{\partial y} = -2y; \quad \left.\frac{\partial z}{\partial y}\right|_P = -2$$

$$-(x-1)(-2) - (y-1)(-2) + (z-2) = 0$$

$$2x - 2 + 2y - 2 + z - 2 = 0$$

$$\Rightarrow \boxed{2x + 2y + z - 6 = 0}$$

15 Puntos