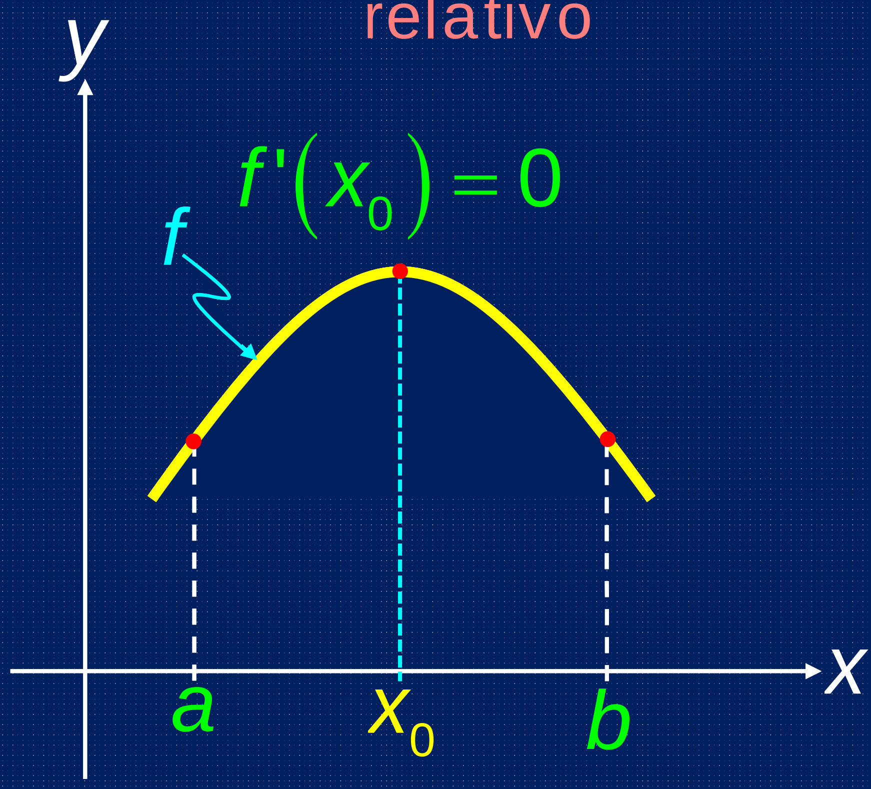
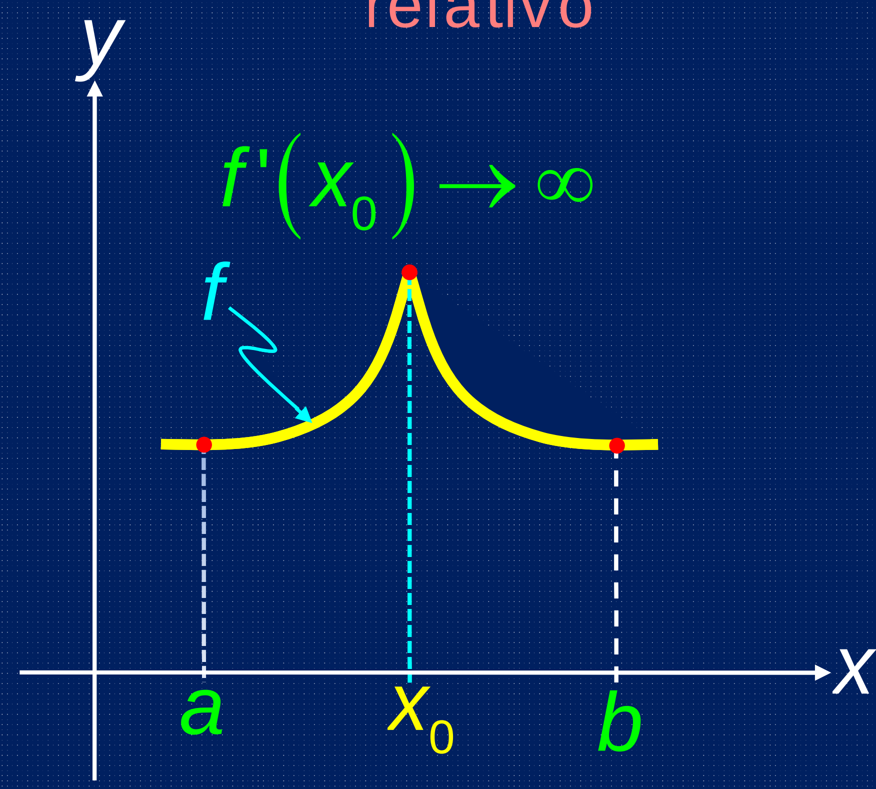
Los “picos” como extremos en el cálculo diferencial de funciones

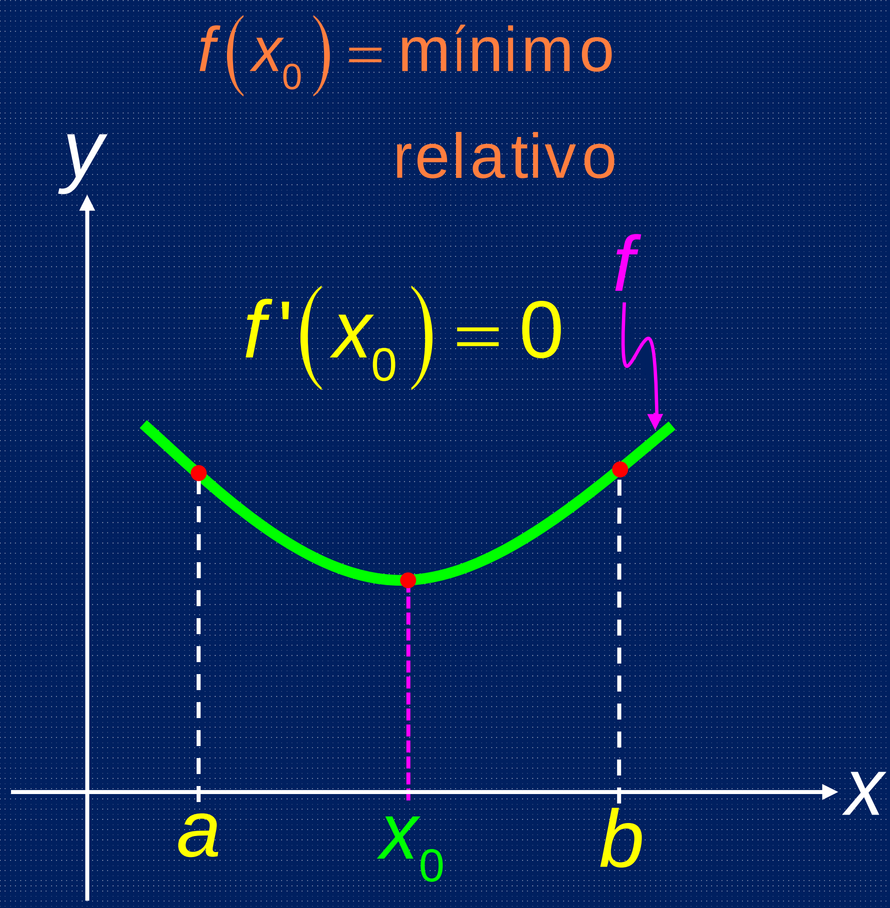
Pablo García y Colomé

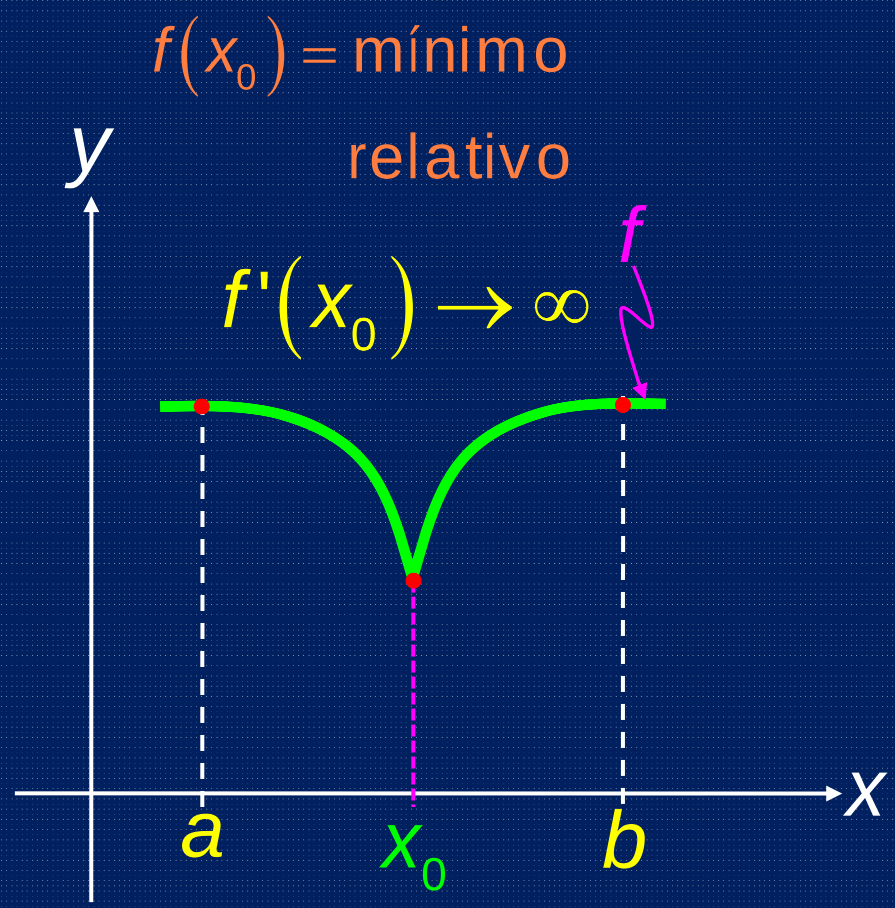
$f(x_0) = \text{máximo relativo}$



$f(x_0) = \text{máximo relativo}$







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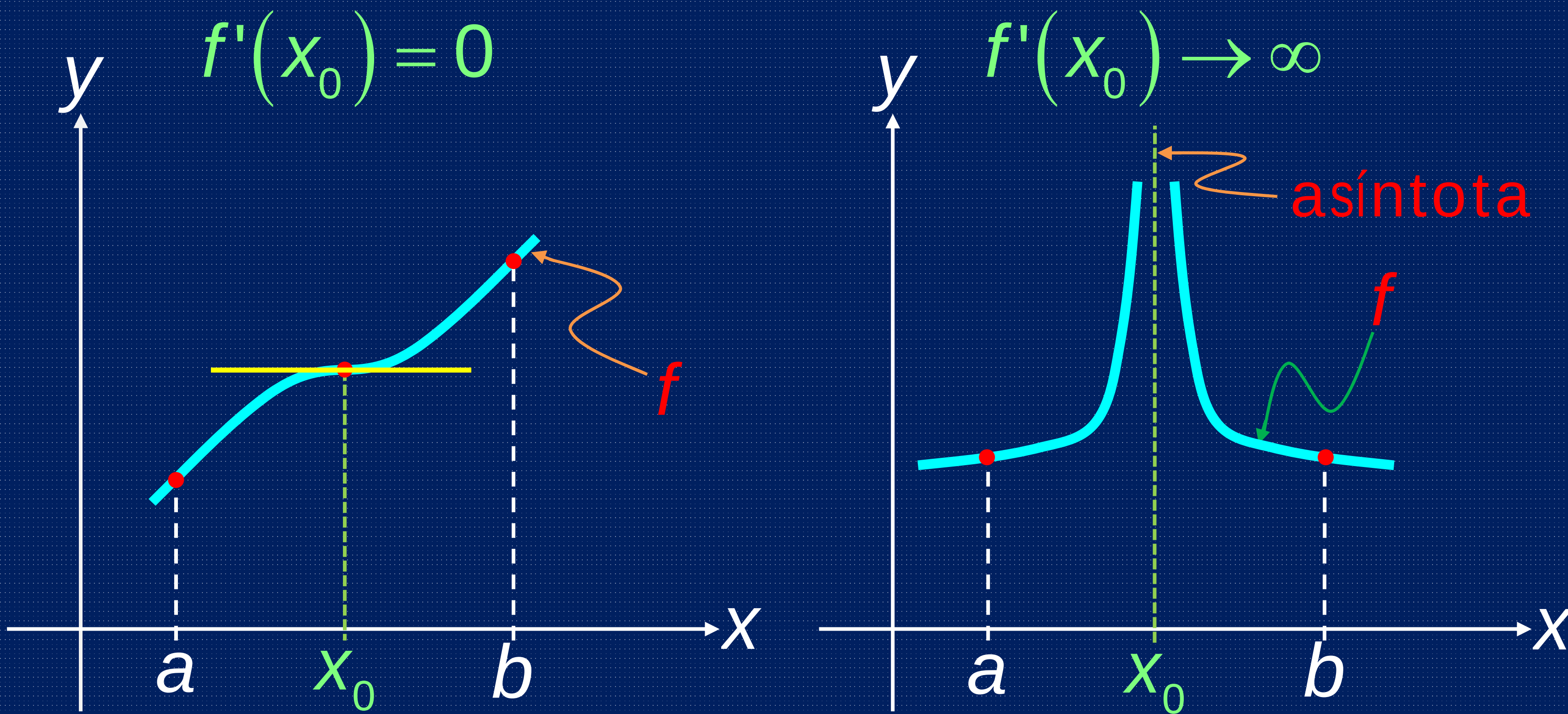
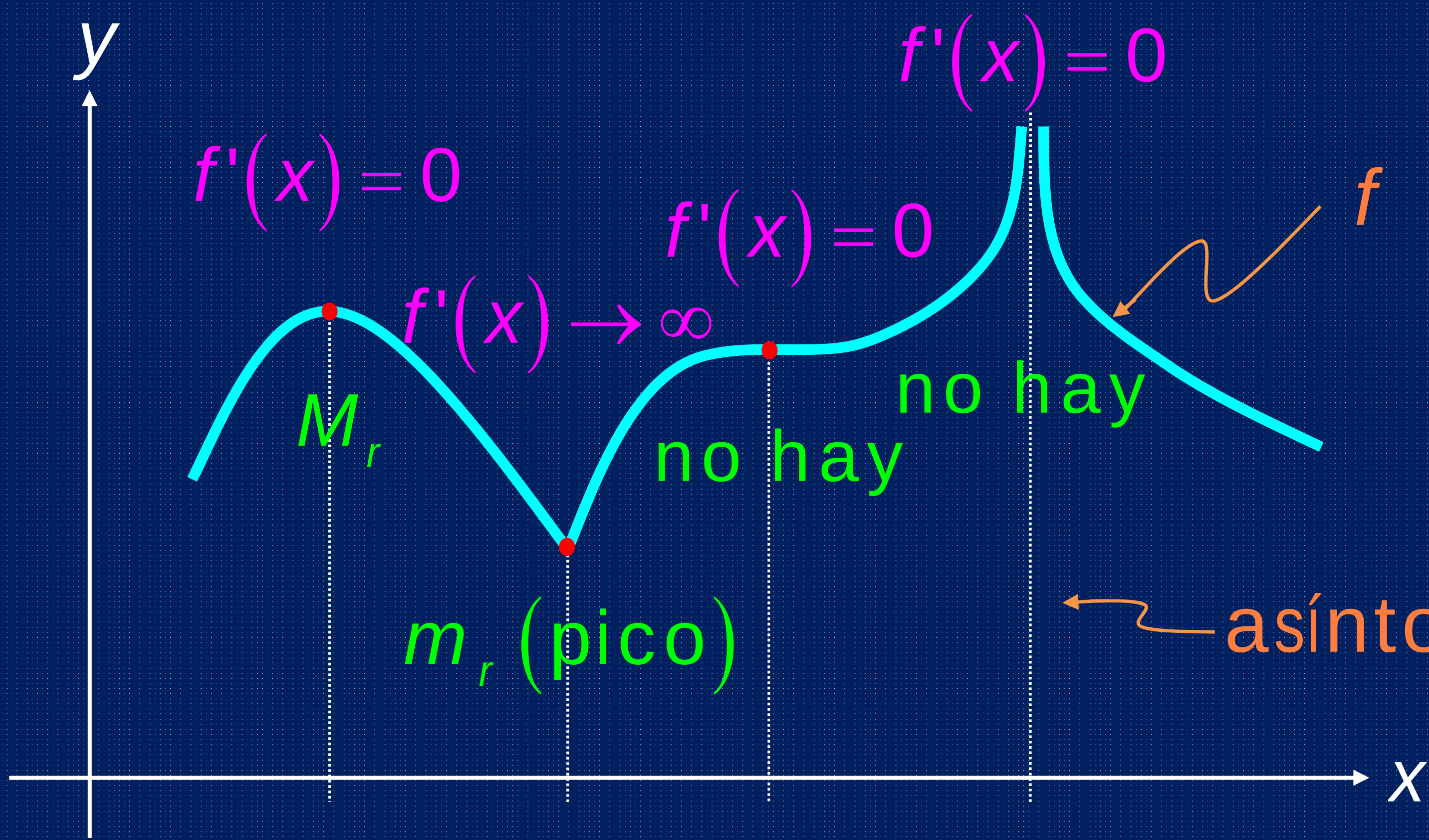


$\therefore$ no hay máximos ni mínimos relativos

DEFINICIÓN. Se conocen como valores críticos de la variable independiente a los valores del eje de las abscisas donde la derivada es cero o no existe



$$f(x) = x + 3x^{\frac{2}{3}}$$

$$f'(x) = 1 + 2x^{-\frac{1}{3}} \Rightarrow \quad f'(x) = \frac{\sqrt[3]{x} + 2}{\sqrt[3]{x}}$$

$$\frac{\sqrt[3]{x} + 2}{\sqrt[3]{x}} = 0 \quad \Rightarrow \quad \sqrt[3]{x} + 2 = 0$$

$$\Rightarrow \quad \sqrt[3]{x} = -2 \quad \Rightarrow \quad x = -8$$

$$\frac{\sqrt[3]{x}+2}{\sqrt[3]{x}} \to \infty \quad \Rightarrow \quad \sqrt[3]{x}=0 \quad \Rightarrow \quad x=0$$

$$x_1=-8 \quad y \quad x_2=0$$

$$x_1 = -8 \quad ; \quad \begin{cases} x = -9 \quad \Rightarrow \quad \dfrac{dy}{dx} = +0.04 > 0 \\ x = -1 \quad \Rightarrow \quad \dfrac{dy}{dx} = -1 < 0 \end{cases}$$

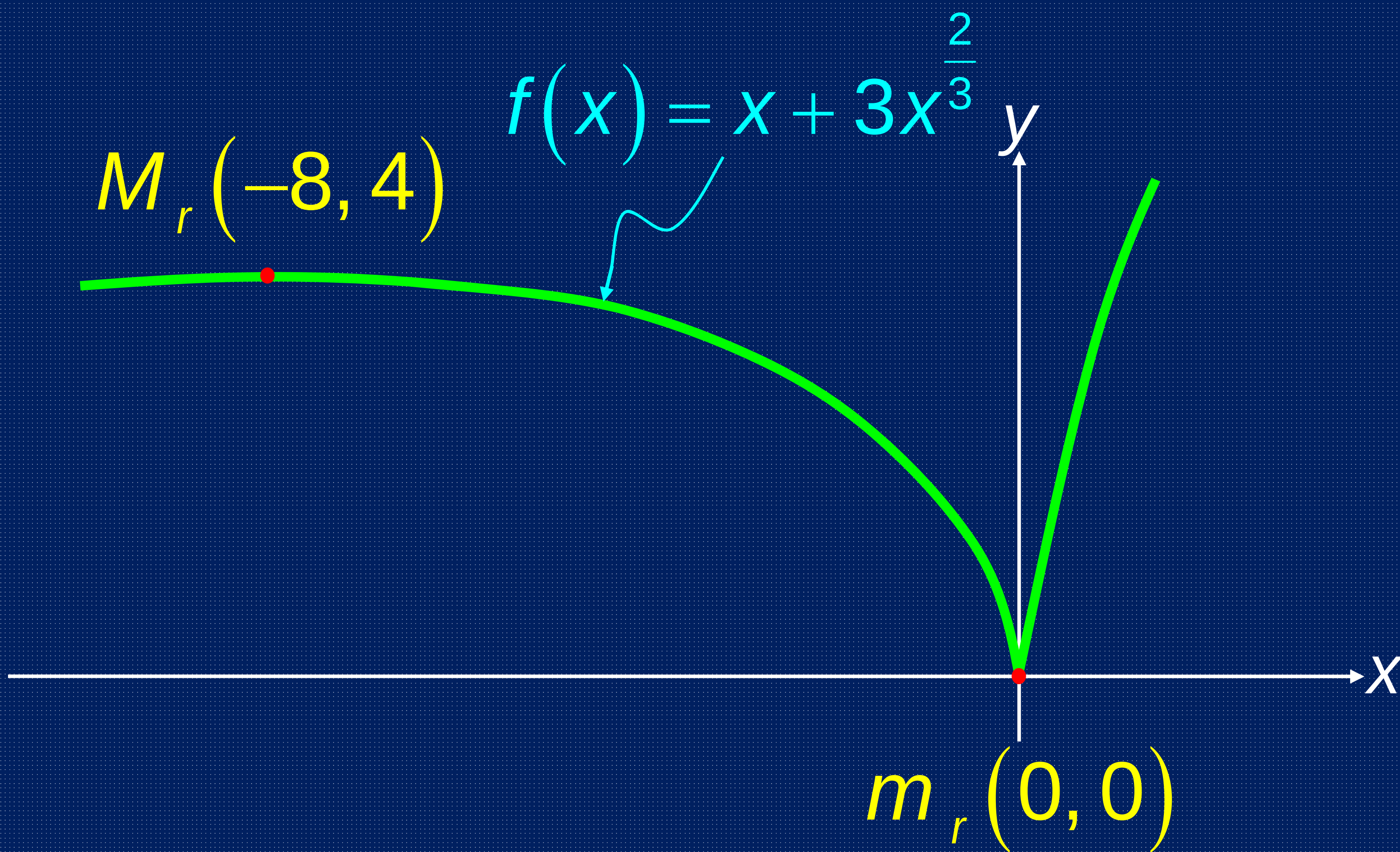
$$x = -8 \quad ; \quad y = -8 + 3\left(-8\right)^{\frac{2}{3}} \quad \Rightarrow \quad y = 4$$

$$\therefore \quad M_r\left(-8, 4\right)$$

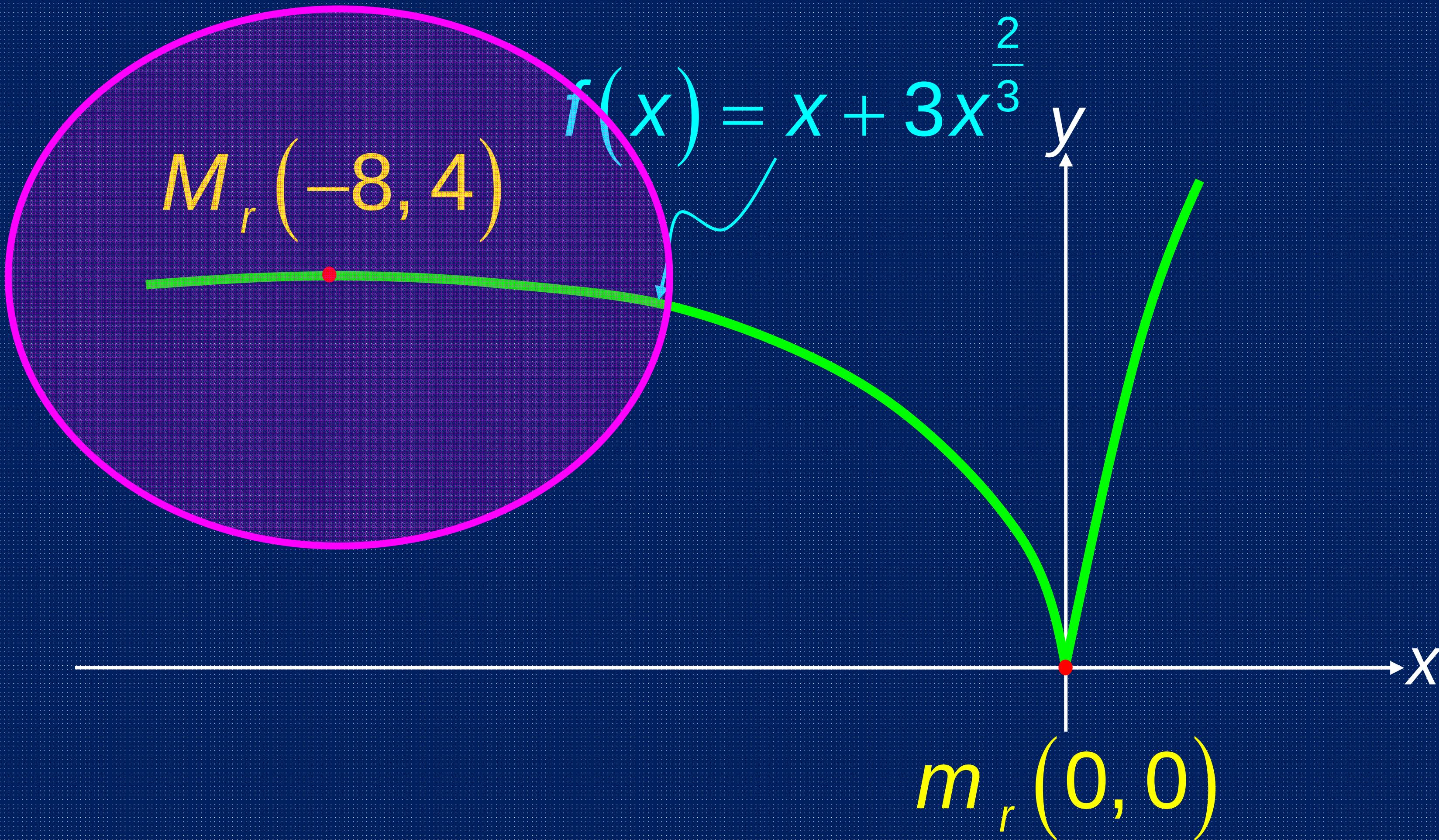
$$x_2 = 0 \quad ; \quad \begin{cases} x = -1 \;\; \Rightarrow \;\; \dfrac{dy}{dx} = -1 < 0 \\[2ex] x = 1 \;\; \Rightarrow \;\; \dfrac{dy}{dx} = +3 > 0 \end{cases}$$

$$x = 0 \quad ; \quad y = 0$$

$$\therefore \quad m_r(0,0)$$



$f(x) = x + 3x^{\frac{2}{3}}$
y
$M_r(-8, 4)$
x
$m_r(0, 0)$



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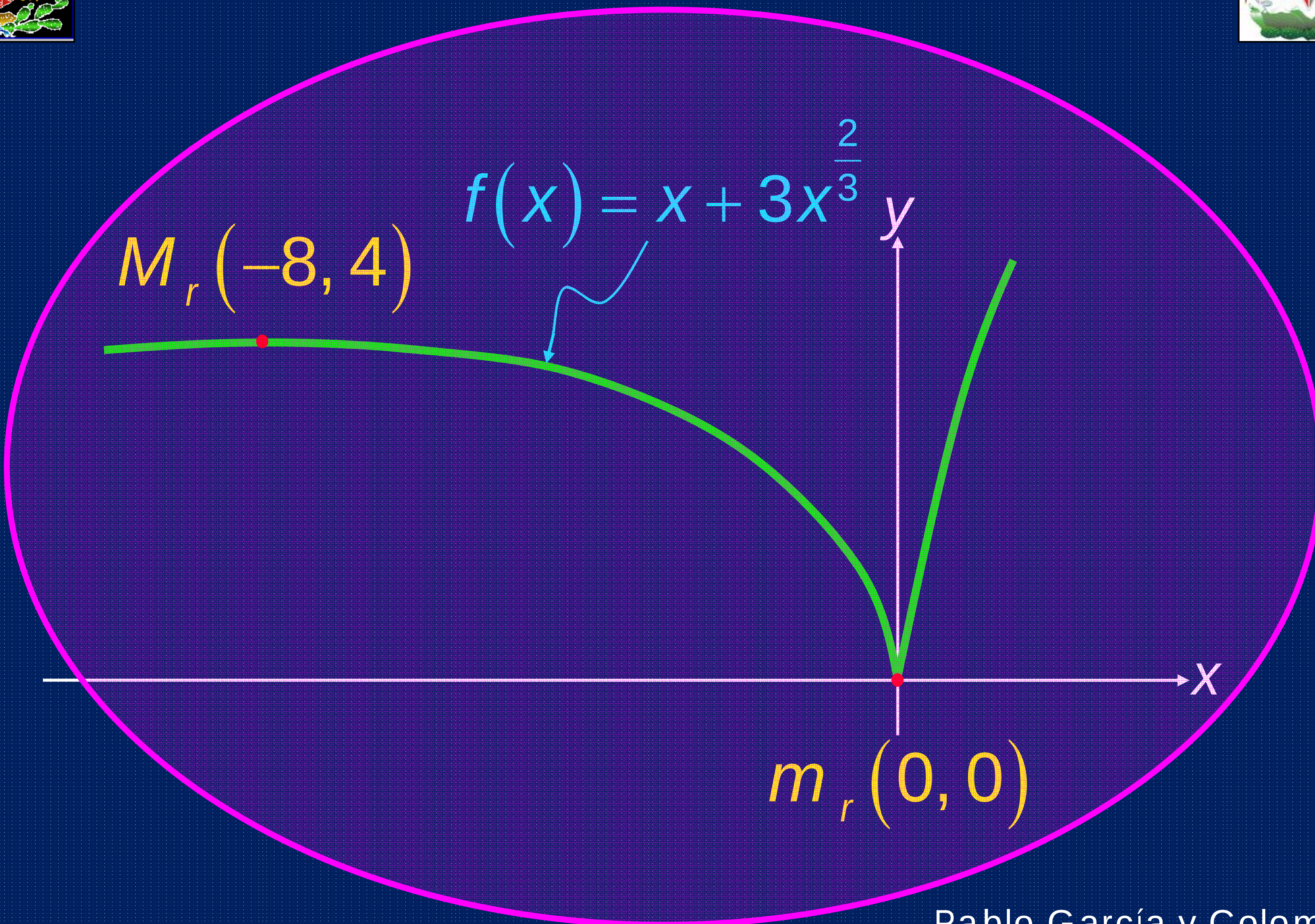

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$$f(x)=\begin{cases}4-\dfrac{x^2}{2} & si \quad x\le 2\\ \dfrac{x}{2}+1 & si \quad x>2\end{cases}$$

$$x<2$$

$$f'(x)=-x \quad ; \quad f'(x)=0 \quad \Rightarrow \quad -x=0$$

$$x=0$$

$$x>2$$

Continuidad en $x = 2$

$$f(2) = 2 = \lim_{x \to 2} f(x)$$

Derivabilidad en $x = 2$

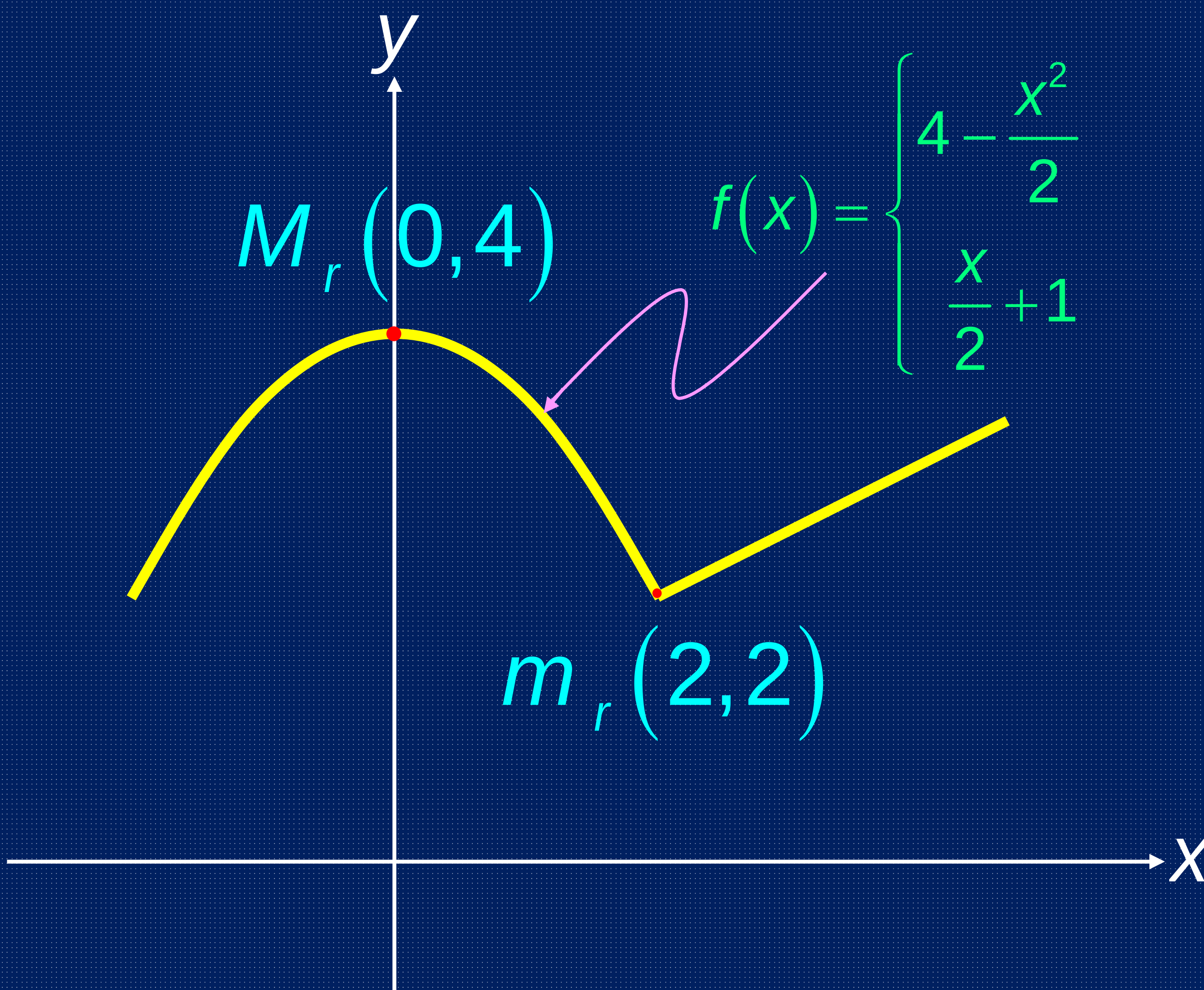
$$f'_{-}(x) = -x\Big|_{x=2} = -2 < 0 \qquad ; \qquad f'_{+}(x) = \frac{1}{2} > 0$$

$$x_1 = 0 \qquad y \qquad x_2 = 2$$

$$x = 0 \quad \Rightarrow \quad \begin{cases} x = -1 \quad \Rightarrow \quad \dfrac{dy}{dx} = 1 > 0 \\ x = 1 \quad \Rightarrow \quad \dfrac{dy}{dx} = -1 < 0 \end{cases} \qquad \therefore \quad M_r(0,4)$$

$$x = 2 \quad \Rightarrow \quad \begin{cases} x = 1 \quad \Rightarrow \quad \dfrac{dy}{dx} = -1 < 0 \\ x = 3 \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2} > 0 \end{cases} \qquad \therefore \quad m_r(2,2)$$

$$f(x)=\begin{cases} 4-\dfrac{x^2}{2} & si \quad x\le 2 \\ \dfrac{x}{2}+1 & si \quad x>2 \end{cases}$$



$$y=\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{3}}\cdot 2x \quad\Rightarrow\quad \frac{dy}{dx}=\frac{x}{\left(x^2-1\right)^{\frac{1}{3}}}$$

$$\frac{dy}{dx}=0 \quad\Rightarrow\quad \frac{x}{\left(x^2-1\right)^{\frac{1}{3}}}=0 \quad\Rightarrow\quad x=0$$

$$\frac{dy}{dx}\to\infty \quad\Rightarrow\quad \left(x^2-1\right)^{\frac{1}{3}}=0 \quad\Rightarrow\quad x=\pm 1$$

$$x_1=-1 \quad ; \quad x_2=0 \quad ; \quad x_3=1$$

$$\frac{dy}{dx}=\frac{x}{\left(x^2-1\right)^{\frac{1}{3}}}$$

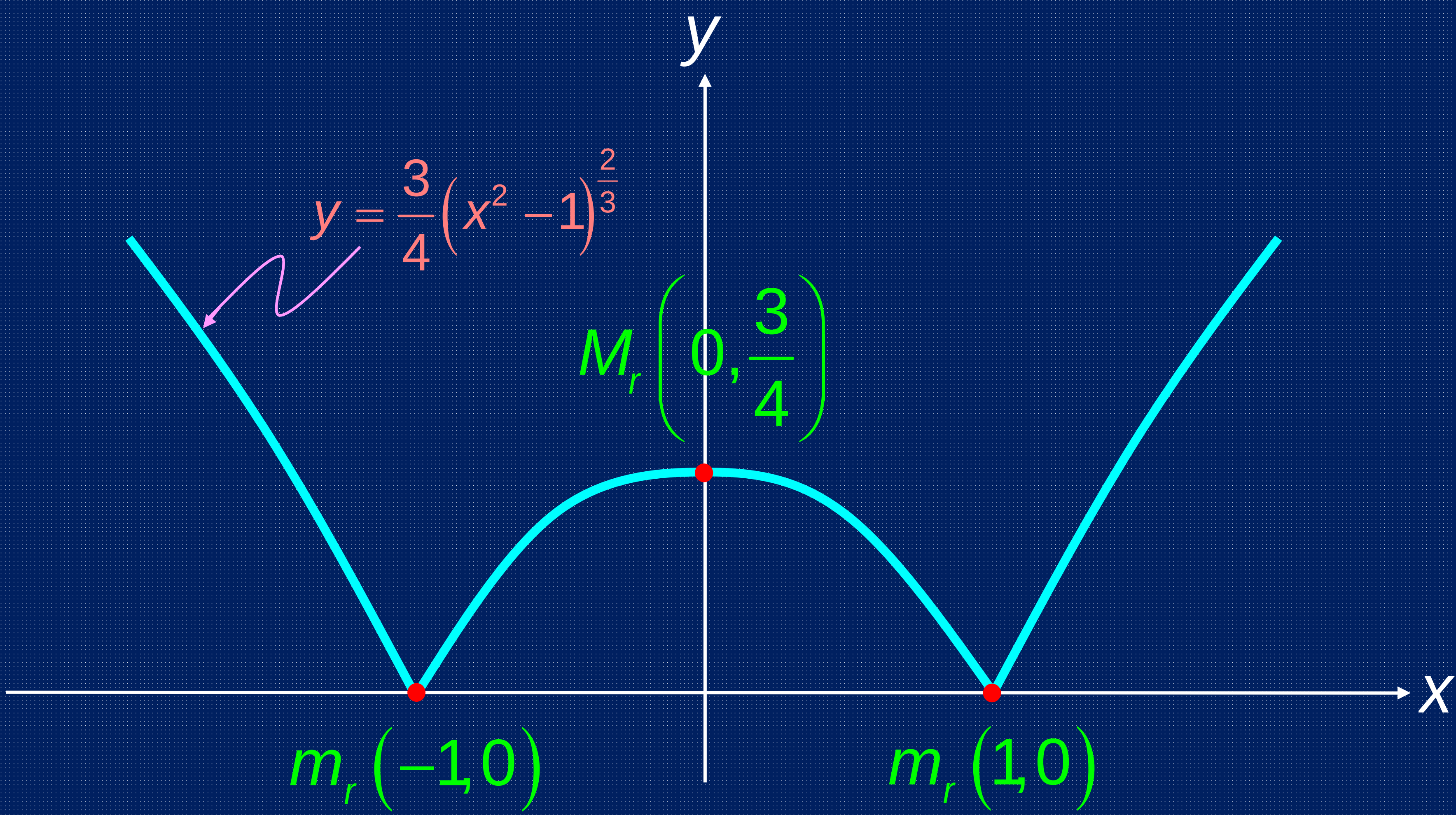
$$x_1=-1\quad\begin{cases} x=-2 & \Rightarrow\quad \dfrac{dy}{dx}<0 \\ x=-0.5 & \Rightarrow\quad \dfrac{dy}{dx}>0 \end{cases}\qquad \therefore\quad m_r\left(-1,0\right)$$

$$\frac{dy}{dx} = \frac{x}{\left(x^2 - 1\right)^{\frac{1}{3}}}$$

$$x_2 = 0 \quad \begin{cases} x = -0.5 \quad \Rightarrow \quad \dfrac{dy}{dx} > 0 \\ x = 0.5 \quad \Rightarrow \quad \dfrac{dy}{dx} < 0 \end{cases} \quad \therefore \quad M_r\left(0, \frac{3}{4}\right)$$

$$\frac{dy}{dx}=\frac{x}{\left(x^2-1\right)^{\frac{1}{3}}}$$

$$x_3=1\quad\begin{cases} x=0.5 & \Rightarrow \quad \dfrac{dy}{dx}<0 \\ x=2 & \Rightarrow \quad \dfrac{dy}{dx}>0 \end{cases}\quad\therefore\quad m_r\left(1,0\right)$$



y
x
$y=\frac{3}{4}\left(x^2-1\right)^{\frac{2}{3}}$
$M_r\left(0,\frac{3}{4}\right)$
$m_r\left(-1,0\right)$
$m_r\left(1,0\right)$

$$y = x^{\frac{5}{3}} - 6x^{\frac{2}{3}}$$

$$\frac{dy}{dx} = \frac{5}{3}x^{\frac{2}{3}} - 4x^{-\frac{1}{3}} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{5x-12}{3x^{\frac{1}{3}}}$$

$$\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{5x-12}{3x^{\frac{1}{3}}} = 0 \quad \Rightarrow \quad 5x-12=0 \quad \Rightarrow \quad x = \frac{12}{5}$$

$$\frac{dy}{dx} \to \infty \quad \Rightarrow \quad \frac{5x-12}{3x^{\frac{1}{3}}} \to \infty \quad \Rightarrow \quad 3x^{\frac{1}{3}} = 0 \quad \Rightarrow \quad x = 0$$

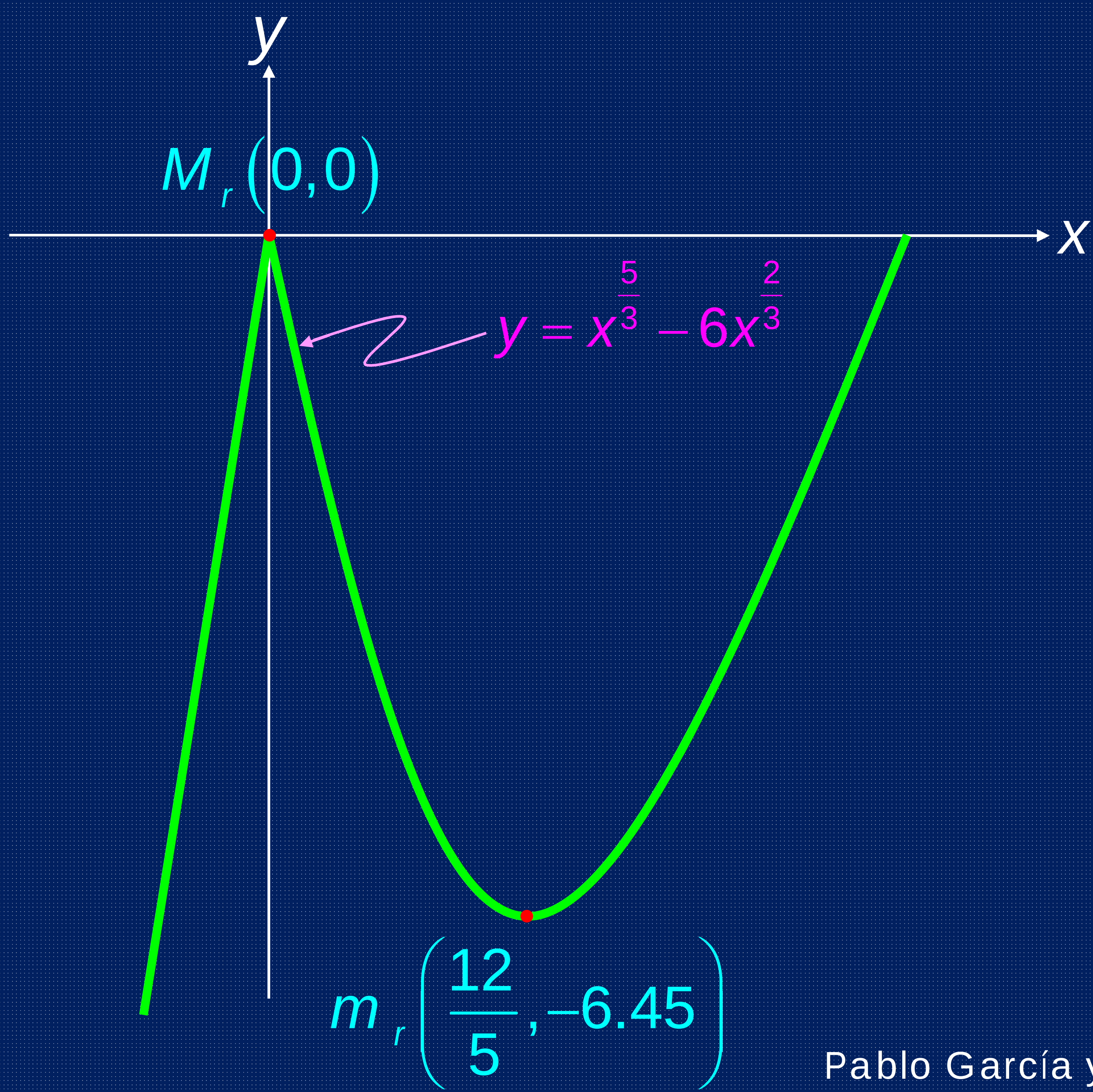
$$x_1 = 0 \quad y \quad x_2 = \frac{12}{5}$$

$$\frac{dy}{dx}=\frac{5x-12}{3x^{\frac{1}{3}}} \quad \Rightarrow \quad \frac{d^2y}{dx^2}=\frac{10x-12}{9x^{\frac{4}{3}}}$$

$$x_1=0 \quad \Rightarrow \quad \frac{d^2y}{dx^2}\to\infty \quad ; \quad \begin{cases} x=-1 \quad \Rightarrow \quad \dfrac{dy}{dx}>0 \\ x=1 \quad \Rightarrow \quad \dfrac{dy}{dx}<0 \end{cases}$$

$$\therefore \quad M_r\left(0,0\right)$$

$$x_2=\frac{12}{5} \quad \Rightarrow \quad \frac{d^2y}{dx^2}>0 \quad \Rightarrow \quad m_r\left(\frac{12}{5},-6.45\right)$$



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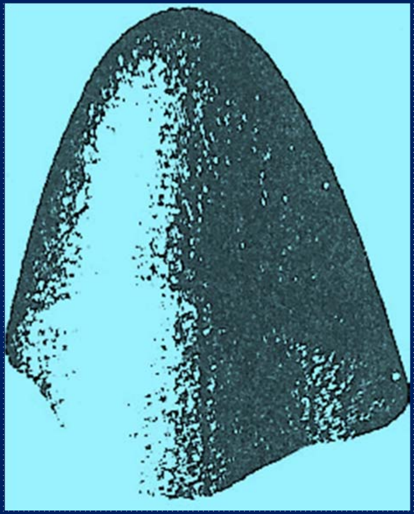
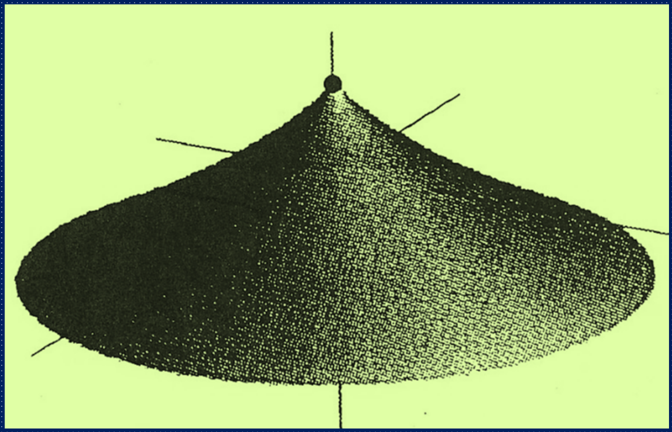
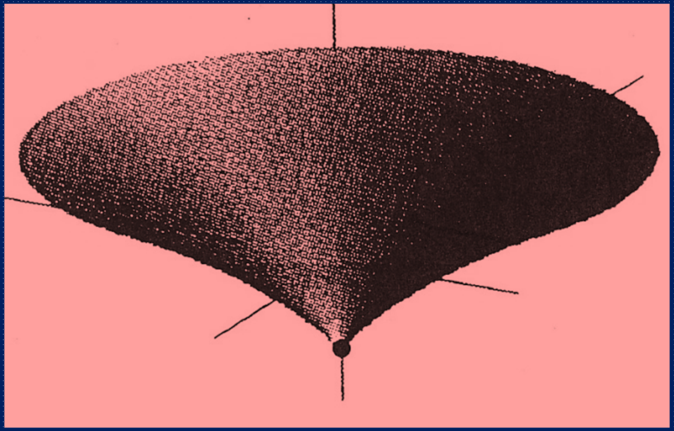
Cálculo vectorial







Sea una función escalar de variable vectorial continua $z = f(x, y)$

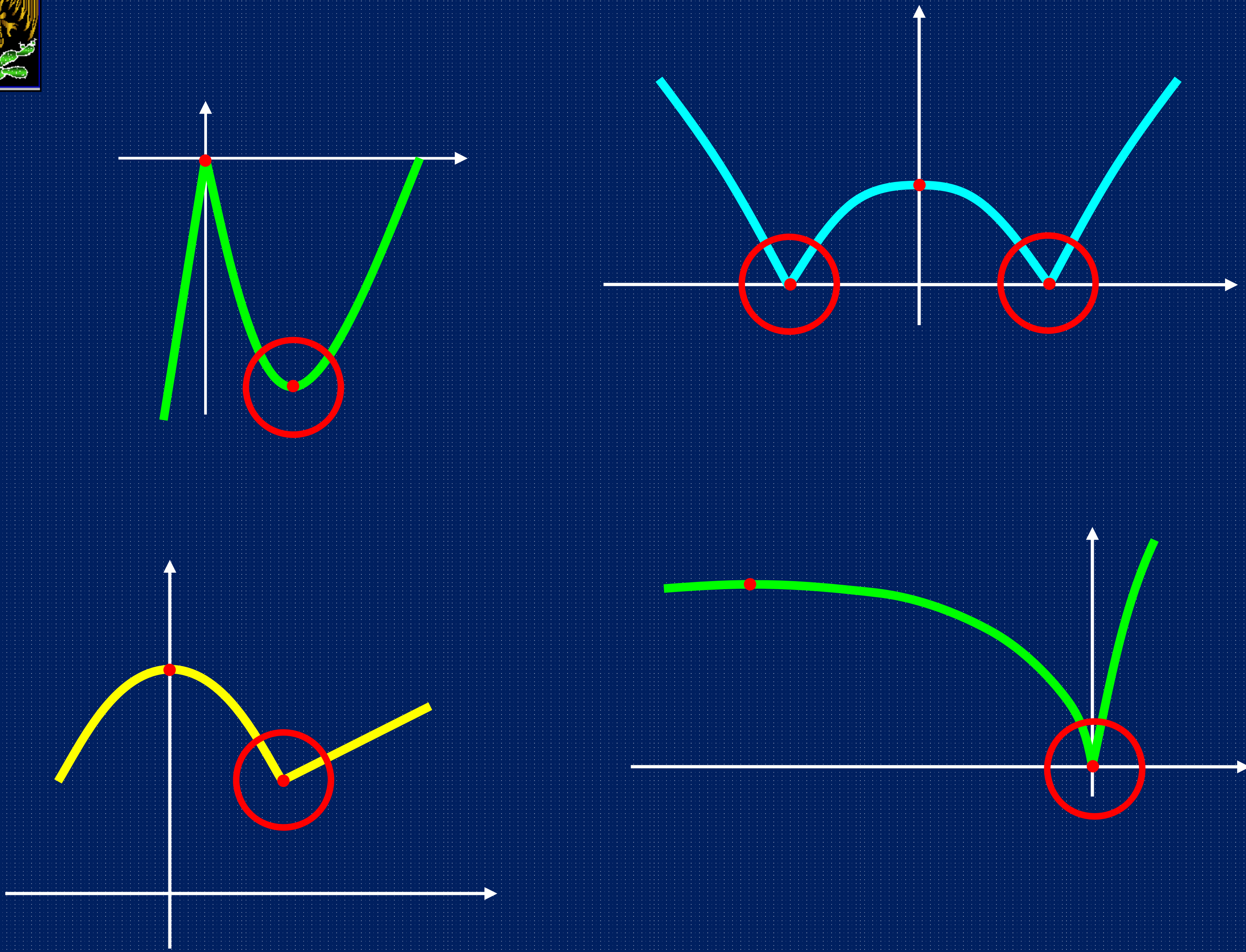
A los puntos donde las primeras derivadas parciales z_x y z_y

Se anulan o no existen, se les denomina puntos críticos

$$z = f(x,y) = (x-1)^{\frac{2}{3}}\left(y^2-4\right)$$

$$\frac{\partial z}{\partial x} = \frac{2\left(y^2-4\right)}{3(x-1)^{\frac{1}{3}}} \qquad y \qquad \frac{\partial z}{\partial y} = 2y(x-1)^{\frac{2}{3}}$$

$$s = \left\{ (1,y,0) \,\middle|\, y \in \mathbb{R} \right\}$$



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Muchas gracias