"Sistemas de ecuaciones diferenciales lineales usando álgebra lineal"

M. I. Yahvé Abdul Ledezma Rubio

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Sistemas de ecuaciones diferenciales de primer orden

$$a2\,x''(t) + a1\,x'(t) + a0\,x(t) = F(t)$$

Normalizando

$$x''(t) + \frac{a1}{a2}\,x'(t) + \frac{a0}{a2}\,x(t) = \frac{F(t)}{a2}$$

$$x'(t) = v(t)$$

$$x''(t) = v'(t)$$

$$x''(t) + \frac{a1}{a2}\, x'(t) + \frac{a0}{a2}\, x(t) = \frac{F(t)}{a2}$$

$$v'(t) + \frac{a1}{a2}\, v(t) + \frac{a0}{a2}\, x(t) = \frac{F(t)}{a2}$$

$$v(t) - x'(t) = 0$$

$$v'(t) + \frac{a1}{a2}\,v(t) + \frac{a0}{a2}\,x(t) = \frac{F(t)}{a2}$$

$$v(t) - x'(t) = 0$$

Por medio de los operadores diferenciales.

$$\begin{bmatrix} D + \dfrac{a1}{a2} & \dfrac{a0}{a2} \\ -1 & D \end{bmatrix} \begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} \dfrac{F(t)}{a2} \\ 0 \end{bmatrix}$$

Desacoplando las ecuaciones

$$\left(D^2 + \frac{a1}{a2}D + \frac{a0}{a2}\right)\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} \dfrac{F(t)}{a2} \\ 0 \end{bmatrix}$$

$$\left(D^2 + \frac{a1}{a2}D + \frac{a0}{a2}\right)x(t) = \begin{bmatrix} D + \frac{a1}{a2} & \frac{F(t)}{a2} \\ -1 & 0 \end{bmatrix}$$

$$\left(D^2 + \frac{a1}{a2}D + \frac{a0}{a2}\right)x(t) = \frac{F(t)}{a2}$$

$$\left(D^2 + \frac{a1}{a2}D + \frac{a0}{a2}\right)v(t) = \begin{bmatrix} \frac{F(t)}{a2} & \frac{a0}{a2} \\ 0 & D \end{bmatrix}$$

$$\left(D^2 + \frac{a1}{a2}D + \frac{a0}{a2}\right)v(t) = D\left[\frac{F(t)}{a2}\right]$$

$$\lambda_{1,2} = -\frac{a1}{2\,a2} \pm \frac{1}{2\,a2}\sqrt{a1^2 - 4\,a0\,a2}$$

$$x(t) = C1\,e^{\lambda 1\,t} + C2\,e^{\lambda 2\,t} + xp(t)$$

$$v(t) = C3\,e^{\lambda 1\,t} + C4\,e^{\lambda 2\,t} + vp(t)$$

Observaciones

- El sistema de ecuaciones se puede descomponer en n ecuaciones diferenciales del mismo orden n de la original.
- Las ecuaciones características y por lo tanto, las raíces y la solución homogénea son similares.
- Al encontrar las soluciones separadas de cada ecuación, aparecen n constantes indeterminadas para cada una de ellas, en total, $2n$ constantes.
- El número total de constantes independientes del sistema es realmente n, por lo que hay que volver a encontrar las relaciones entre las soluciones del sistema de ecuaciones.

$$x'(t) = \lambda 1\, C1\, e^{\lambda 1\, t} + \lambda 2\, C2\, e^{\lambda 2\, t} + D[xp(t)]$$

$$\lambda 1\, C1\, e^{\lambda 1\, t} + \lambda 2\, C2\, e^{\lambda 2\, t} + D[xp(t)] = C3\, e^{\lambda 1\, t} + C4\, e^{\lambda 2\, t} + vp(t)$$

$$C3 = \lambda 1\, C1$$

$$C4 = \lambda 2\, C2$$

Forma matricial de un sistema de ecuaciones

$$v'(t) + \frac{a1}{a2}\, v(t) + \frac{a0}{a2}\, x(t) = \frac{F(t)}{a2}$$

$$x'(t) - v(t) = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v'(t) \\ x'(t) \end{bmatrix} + \begin{bmatrix} a\,1/a\,2 & a\,0/a\,2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ x(t) \end{bmatrix}$$

$$= \begin{bmatrix} F(t\,)/a\,2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v'(t) \\ x'(t) \end{bmatrix} + \begin{bmatrix} a\,1/a\,2 & a\,0/a\,2 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} F(t\,)/a\,2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v'(t) \\ x'(t) \end{bmatrix} = \begin{bmatrix} -\frac{a1}{a2} & -\frac{a0}{a2} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ x(t) \end{bmatrix} + \begin{bmatrix} F\,(t)/a\,2 \\ 0 \end{bmatrix}$$

$$\bar{q}' = A\bar{q} + \bar{b}$$

$$\bar{q}' = A\bar{q} + \bar{b}$$

$$x' = P\,x + g(t)$$

$$x' - P\,x = g(t)$$

$$\mu = e^{-\int P\,dt}$$

$$\mu(x' - P\,x) = \mu\,g(t)$$

$$e^{-\int P\,dt}(x' - P\,x) = e^{-\int P\,dt}\,g(t)$$

$$e^{-\int P\,dt}x' - e^{-\int P\,dt}P\,x = e^{-\int P\,dt}\,g(t)$$

$$d\left[e^{\int -P\,dt}x\right] = e^{\int -P\,dt}\,g(t)dt$$

$$e^{-\int P\,dt}x' - e^{-\int P\,dt}P\,x = e^{-\int P\,dt}\,g(t)$$

$$d\left[e^{\int -P\,dt}x\right] = e^{\int -P\,dt}\,g(t)dt$$

$$e^{-\int P\,dt}x = \int e^{-\int P\,dt}\,g(t)dt + C1$$

$$x = e^{\int P\,dt}\int e^{-\int P\,dt}\,g(t)dt + C1e^{\int P\,dt}$$

Valores propios, vectores propios

$$A-\lambda I=\begin{bmatrix}-\frac{a1}{a2} & -\frac{a0}{a2}\\ 1 & 0\end{bmatrix}-\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$

$$A-\lambda I=\begin{bmatrix}-\frac{a1}{a2}-\lambda & -\frac{a0}{a2}\\ 1 & -\lambda\end{bmatrix}$$

$$Det(A-\lambda I)=\left(-\frac{a1}{a2}-\lambda\right)(-\lambda)-\left(-\frac{a0}{a2}\right)=0$$

$$\lambda^2+\frac{a1}{a2}\lambda+\frac{a0}{a2}=0$$

$$(A - \lambda 1\, I)\begin{bmatrix} k11 \\ k21 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{a1}{a2} - \lambda 1 & -\frac{a0}{a2} \\ 1 & -\lambda 1 \end{bmatrix}\begin{bmatrix} k11 \\ k21 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$k11 - \lambda 1\, k21 = 0$$

$$k21 = 1$$

$$k11 = \lambda 1$$

$$K1 = \begin{bmatrix} \lambda 1 \\ 1 \end{bmatrix}$$

$$(A - \lambda 2\, I) \begin{bmatrix} k12 \\ k22 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{a1}{a2} - \lambda 2 & -\frac{a0}{a2} \\ 1 & -\lambda 2 \end{bmatrix} \begin{bmatrix} k12 \\ k22 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$k12 - \lambda 2\, k22 = 0$$

$$k22 = 1$$

$$k12 = \lambda 2$$

$$K2 = \begin{bmatrix} \lambda 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = C1\,K1\,e^{\lambda 1\,t} + C2\,K2\,e^{\lambda 2\,t}$$

$$\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = C1\,\begin{bmatrix} \lambda 1 \\ 1 \end{bmatrix} e^{\lambda 1\,t} + C2\,\begin{bmatrix} \lambda 2 \\ 1 \end{bmatrix} e^{\lambda 2\,t}$$

$$\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = C1\,\begin{bmatrix} \lambda 1 e^{\lambda 1\,t} \\ e^{\lambda 1\,t} \end{bmatrix} + C2\,\begin{bmatrix} \lambda 2\, e^{\lambda 2\,t} \\ e^{\lambda 2\,t} \end{bmatrix}$$

$$X1 = \begin{bmatrix} \lambda 1 e^{\lambda 1\, t} \\ e^{\lambda 1\, t} \end{bmatrix} = K1\, e^{\lambda 1\, t}$$

$$X2 = \begin{bmatrix} \lambda 2\, e^{\lambda 2\, t} \\ e^{\lambda 2\, t} \end{bmatrix} = K2\, e^{\lambda 2\, t}$$

Wronskiano

$$W(X1, X2) = Det[X1 \;\; X2]$$

$$W(X1, X2) = Det\begin{bmatrix} \lambda 1 e^{\lambda 1\, t} & \lambda 2\, e^{\lambda 2\, t} \\ e^{\lambda 1\, t} & e^{\lambda 2\, t} \end{bmatrix}$$

$$W(X1, X2) = \lambda 1 e^{\lambda 1\, t} e^{\lambda 2\, t} - \lambda 2\, e^{\lambda 2\, t} e^{\lambda 1\, t}$$

$$W(X1, X2) = (\lambda 1 - \lambda 2)\, e^{(\lambda 1 + \lambda 2)t}$$

$$\Phi(\mathrm{t}) = [X1 \;\; X2]$$

$$\Phi(\mathrm{t}) = \left[K1\, e^{\lambda 1\, t} \;\; K2\, e^{\lambda 2\, t}\right]$$

$$\Phi(\mathrm{t}) = \begin{bmatrix} k11\, e^{\lambda 1\, t} & k12\, e^{\lambda 2\, t} \\ k12\, e^{\lambda 1\, t} & k22\, e^{\lambda 2\, t} \end{bmatrix}$$

$$\Phi(\mathrm{t}) = \begin{bmatrix} k11 & k12 \\ k12 & k22 \end{bmatrix} \begin{bmatrix} e^{\lambda 1\, t} \\ e^{\lambda 2\, t} \end{bmatrix}$$

$$\begin{bmatrix} v(t) \\ x(t) \end{bmatrix} = \begin{bmatrix} k11\, e^{\lambda 1\, t} & k12\, e^{\lambda 2\, t} \\ k12\, e^{\lambda 1\, t} & k22\, e^{\lambda 2\, t} \end{bmatrix} \begin{bmatrix} C1 \\ C2 \end{bmatrix}$$

$$\overline{qSol} = \Phi(\mathrm{t})\, \bar{C}$$

$$\bar{q}' = A\bar{q}$$

$$\bar{q} = \Phi(\mathrm{t})\ \bar{C}$$

$$\bar{q}' = \Phi'(\mathrm{t})\ \bar{C}$$

$$\bar{q}' = A\bar{q}$$

$$\Phi'(\mathrm{t})\bar{C} = A\ \Phi(\mathrm{t})\ \bar{C}$$

$$\Phi'(\mathrm{t})\bar{C} - A\ \Phi(\mathrm{t})\bar{C} = \bar{0}$$

$$\Phi'(\mathrm{t}) - A\,\Phi(\mathrm{t}) = \bar{0}$$

$$\Phi'(\mathrm{t}) = A\,\Phi(\mathrm{t})$$

Valores propios reales diferentes

$$3\,x'(t) - \ 3\,x(t) - 9\,y(t) = 0$$

$$-10\,x(t) + 2\,y'(t) - 6\,y(t) = 0$$

$$x'(t) = \ x(t) + 3\,y(t)$$

$$y'(t) = 5\,x(t) + 3\,y(t)$$

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$Det(A - \lambda\, I) = 0$$

$$Det \begin{bmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 15 = 0$$

$$\lambda^2 - 4\lambda - 12 = 0$$

$$\lambda 1 = -2$$

$$\lambda 2 = 6$$

$$(A - \lambda 1\ I)K1 = \bar{0}$$

$$\begin{bmatrix} 1+2 & 3 \\ 5 & 3+2 \end{bmatrix} \begin{bmatrix} k11 \\ k21 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$k11 + k21 = 0$$

$$k11 = 1$$

$$k21 = -1$$

$$K1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(A - \lambda 2\ I)K2 = \bar{0}$$

$$\begin{bmatrix} 1-6 & 3 \\ 5 & 3-6 \end{bmatrix} \begin{bmatrix} k12 \\ k22 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5\ k12 - 3k22 = 0$$

$$k11 = 3$$

$$k21 = 5$$

$$K2 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = C1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2\,t} + C2 \begin{bmatrix} 3 \\ 5 \end{bmatrix} e^{6\,t}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = C1 \begin{bmatrix} e^{-2\,t} \\ -e^{-2\,t} \end{bmatrix} + C2 \begin{bmatrix} 3e^{6\,t} \\ 5e^{6\,t} \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^{-2\,t} & 3e^{6\,t} \\ -e^{-2\,t} & 5e^{6\,t} \end{bmatrix} \begin{bmatrix} C1 \\ C2 \end{bmatrix}$$

Valores propios reales repetidos

a) Si con un valor repetido n veces se pueden encontrar n vectores linealmente independientes, la solución será la combinación de la misma función con los vectores independientes.

b) si el valor principal de multiplicidad n solamente tiene un vector propio, las soluciones se construyen a partir de nuevos vectores independientes.

$$X'(t) = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} X(t)$$

$$X(t) = \begin{bmatrix} x1(t) \\ x2(t) \\ x3(t) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

$$Det(A - \lambda I) = 0$$

$$-(-5 + \lambda)(1 + \lambda)^2 = 0$$

$$\begin{bmatrix} 1+1 & -2 & 2 \\ -2 & 1+1 & -2 \\ 2 & -2 & 1+1 \end{bmatrix} \begin{bmatrix} k11 \\ k21 \\ k31 \end{bmatrix} = \bar{0}$$

$$k11 - k21 + k31 = 0$$

$$K1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad K2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1-5 & -2 & 2 \\ -2 & 1-5 & -2 \\ 2 & -2 & 1-5 \end{bmatrix} \begin{bmatrix} k13 \\ k23 \\ k33 \end{bmatrix} = \bar{0}$$

$$k23 + k33 = 0$$

$$k13 - k33 = 0$$

$$K3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$X(t) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} e^{5t}$$

$$X1 = K\,e^{\lambda t}$$

$$X2 = K\,t\,e^{\lambda t} + P\,e^{\lambda t}$$

$$X3 = K\,\frac{t^2}{2}\,e^{\lambda t} + P\,t\,e^{\lambda t} + Q\,e^{\lambda t}$$

$$X2 = K\,t\,e^{\lambda t} + P\,e^{\lambda t}$$

$$X2'(t) = A\,X2(t)$$

$$K\lambda\;t\,e^{\lambda t} + K\,e^{\lambda t} + P\,\lambda\;e^{\lambda t} =$$

$$= A\left(K\,t\,e^{\lambda t} + P\,e^{\lambda t}\right)$$

$$(A - \lambda I)K = \bar{0}$$

$$(A - \lambda I)P = K$$

$$(A - \lambda I)K = \bar{0}$$

$$(A - \lambda I)P = K$$

$$(A - \lambda I)Q = P$$

$$Xn = K \frac{t^{n-1}}{(n-1)!} e^{\lambda t} + P \frac{t^{n-2}}{(n-2)!} e^{\lambda t} + \cdots$$

$$x'(t) = 3x - y - z$$

$$y'(t) = x + y - z$$

$$z'(t) = x - y + z$$

$$X' = \begin{bmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} X$$

$$\lambda 1 = 1$$

$$\lambda 2 = 2 \qquad \lambda 3 = 2$$

$$K1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad K22 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$K21 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$X1 = K1 e^{\lambda 1\ t}$$

$$X2 = K21\ e^{\lambda 2\ t}$$

$$X3 = K22\ e^{\lambda 2\ t}$$

$$X = C1\begin{bmatrix}1\\1\\1\end{bmatrix}e^{t} + C2\begin{bmatrix}1\\0\\1\end{bmatrix}e^{2\,t} +$$

$$+ C3\begin{bmatrix}1\\1\\0\end{bmatrix}e^{2\,t}$$

$$X' = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix} X$$

$$\lambda 1 = 4$$

$$\lambda 2 = 4 \qquad \lambda 3 = 4$$

$$\begin{bmatrix} 4-4 & 1 & 0 \\ 0 & 4-4 & 1 \\ 0 & 0 & 4-4 \end{bmatrix} \begin{bmatrix} k1 \\ k2 \\ k3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} k1 \\ k2 \\ k3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad K = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4-4 & 1 & 0 \\ 0 & 4-4 & 1 \\ 0 & 0 & 4-4 \end{bmatrix} \begin{bmatrix} p1 \\ p2 \\ p3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p1 \\ p2 \\ p3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$p3 = 0, \qquad p2 = 1, \qquad p1 = 1$$

$$P = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4-4 & 1 & 0 \\ 0 & 4-4 & 1 \\ 0 & 0 & 4-4 \end{bmatrix} \begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q1 \\ q2 \\ q3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$q3 = 1, \qquad q2 = 1, \qquad q1 = 1$$

$$Q = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$X1 = K\, e^{\lambda 1\, t}$$

$$X2 = K\, t\, e^{\lambda 2\, t} + P e^{\lambda 2\, t}$$

$$X3 = K\, \frac{t^2}{2} e^{\lambda 2\, t} + P\, t\, e^{\lambda 2\, t} + Q e^{\lambda 2\, t}$$

$$X = C1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e^{4\,t} +$$

$$+C2 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right) e^{4\,t}$$

$$+C3 \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{t^2}{2} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) e^{4\,t}$$

Valores propios imaginarios conjugados

$$A = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$$

$$(a11 - \lambda)(a22 - \lambda) - a12\ a21 = 0$$

$$\lambda_{1,2} = \alpha \pm \beta\, i$$

$$(A - \lambda 1\, I)K1 = \bar{0}$$

$$\begin{bmatrix} a11 - \alpha - \beta\, i & a12 \\ a21 & a22 - \alpha - \beta\, i \end{bmatrix} \begin{bmatrix} k11 \\ k21 \end{bmatrix} = \bar{0}$$

$$k11(a11 - \alpha - \beta\, i) + a12\, k21 = 0$$

$$k11 = 1,$$

$$k21 = \frac{(-a11 + \alpha + \beta\, i)}{a12}$$

$$K1 = \begin{bmatrix} 1 \\ \frac{(-a11 + \alpha + \beta\, i)}{a12} \end{bmatrix}$$

$$(A - \lambda 2\, I)K2 = \bar{0}$$

$$\begin{bmatrix} a11 - \alpha + \beta\ i & a12 \\ a21 & a22 - \alpha + \beta\ i \end{bmatrix} \begin{bmatrix} k12 \\ k22 \end{bmatrix} = \bar{0}$$

$$k12(a11 - \alpha + \beta\ i) + a12\ k22 = 0$$

$$k12 = 1$$

$$k22 = \frac{(-a11 + \alpha - \beta\, i)}{a12}$$

$$K2 = \begin{bmatrix} 1 \\ \frac{(-a11 + \alpha - \beta\, i)}{a12} \end{bmatrix}$$

$$K1 = \left[\frac{1}{\frac{(-a11 + \alpha + \beta\, i)}{a12}} \right]$$

$$K2 = \left[\frac{1}{\frac{(-a11 + \alpha - \beta\, i)}{a12}} \right]$$

$$\lambda_1 = \overline{\lambda_2}$$

$$K1 = \overline{K2}$$

$$X1 = K1\, e^{(\alpha+\beta i)t}$$

$$X2 = K2\, e^{(\alpha-\beta i)t} = \overline{K1} e^{\overline{(\alpha+\beta i)t}}$$

$$Xh = C1\, X1 + C2\, X2$$

$$Xh = C1\, K1\, e^{(\alpha+\beta i)t} + C2\, K2\, e^{(\alpha-\beta i)t}$$

$$B1 = \frac{1}{2}\left(K1 + \overline{K1}\right), \qquad B1 = Re[K1]$$

$$B2 = \frac{i}{2}\left(K1 - \overline{K1}\right), \qquad B2 = -Im[K1]$$

$$X1n = e^{\alpha t}\left(B1 \cos\beta\, t + B2 \sin\beta\, t\right)$$

$$X2n = e^{\alpha t}\left(B2 \cos\beta\, t - B1 \sin\beta\, t\right)$$

$$x'(t) = 6\,x(t) - \,y(t)$$

$$y'(t) = 5\,x(t) + 2\,y(t)$$

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix}$$

$$Det \begin{bmatrix} 6-\lambda & -1 \\ 5 & 2-\lambda \end{bmatrix} = 0$$

$$\lambda^2 - 8\lambda + 17 = 0$$

$$\lambda 1 = 4 + i \qquad \lambda 2 = 4 - i$$

$$\begin{bmatrix} 6-4-i & -1 \\ 5 & 2-4-i \end{bmatrix} \begin{bmatrix} k11 \\ k21 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5\,k11 + (-2-i)k21 = 0$$

$$k11 = 2 + i$$

$$k21 = 5$$

$$K1 = \begin{bmatrix} 2+i \\ 5 \end{bmatrix}$$

$$B1 = \frac{1}{2}(K1 + \overline{K1}), \qquad B1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$B2 = \frac{i}{2}(K1 - \overline{K1}), \qquad B2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$X1n = e^{\alpha t}\left(B1 \cos\beta\, t + B2 \sin\beta\, t\right)$$

$$X2n = e^{\alpha t}\left(B2 \cos\beta\, t - B1 \sin\beta\, t\right)$$

$$X1n = e^{4t}\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix} \cos\beta\, t + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \sin\beta\, t\right)$$

$$X2n = e^{4t}\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix} \cos\beta\, t - \begin{bmatrix} 2 \\ 5 \end{bmatrix} \sin\beta\, t\right)$$

Parte no homogénea

$$x' = P\,x + g(t)$$

$$x = e^{\int P\,dt} \int e^{-\int P\,dt}\, g(t)dt + C1e^{\int P\,dt}$$

$$X' = AX + F(t)$$

$$\Phi(t) = [X1 \;\; X2 \;\; \cdots]$$

$$XSol = \Phi(t)\,\bar{C}$$

$$Xp = \Phi(t)\,U(t)$$

$$U(t) = \begin{bmatrix} u1(t) \\ u2(t) \\ \vdots \end{bmatrix}$$

$$\Phi(\mathrm{t})U'(t) = F(t)$$

$$U'(t) = \Phi^{-1}(\mathrm{t})F(t)$$

$$U(t) = \int \Phi^{-1}(\mathrm{t})F(t)\ dt + \bar{C}$$

$$XSol = \Phi(t)\,U(t)$$

$$XSol = Xp + Xh$$

$$x = e^{\int P\,dt} \int e^{-\int P\,dt}\, g(t)dt + C1e^{\int P\,dt}$$

$$XSol = \Phi(\mathrm{t})\,\bar{C} + \Phi(\mathrm{t}) \int \Phi^{-1}(\mathrm{t})\,F(t)\;dt$$

$$X' = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} X + \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^t$$

$$(-\lambda)(3-\lambda)+2=0$$

$$\lambda 1 = 1$$

$$\lambda 2 = 2$$

$$K1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$K2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Phi(t) = [X1\ X2]$$

$$\Phi(t) = \begin{bmatrix} 2e^{t} & 1e^{2t} \\ 1e^{t} & 1e^{2t} \end{bmatrix}$$

$$\Phi^{-1}(t) = \frac{1}{e^{3t}} \begin{bmatrix} e^{2t} & -e^{2t} \\ -e^{t} & 2e^{t} \end{bmatrix}$$

$$U(t) = \int \begin{bmatrix} e^{-t} & -e^{-t} \\ -e^{-2t} & 2e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t}\, dt$$

$$= \int \begin{bmatrix} 2 \\ -3e^{-t} \end{bmatrix} dt$$

$$= \begin{bmatrix} 2t \\ 3e^{-t} \end{bmatrix} + \begin{bmatrix} C1 \\ C2 \end{bmatrix}$$

$$XSol = \Phi(t)\,U(t)$$

$$XSol = \begin{bmatrix} 2e^{t} & e^{2t} \\ e^{t} & e^{2t} \end{bmatrix} \begin{bmatrix} 2t \\ 3e^{-t} \end{bmatrix} + [$$

$$+ \begin{bmatrix} 2e^{t} & e^{2t} \\ e^{t} & e^{2t} \end{bmatrix} \begin{bmatrix} C1 \\ C2 \end{bmatrix}$$

$$XSol = C1\begin{bmatrix}2\\1\end{bmatrix}e^{t} + C2\begin{bmatrix}1\\1\end{bmatrix}e^{2t} +$$

$$+\begin{bmatrix}4te^{t} + 3e^{t}\\2te^{t} + 3e^{t}\end{bmatrix}$$